

# The Effect of Sensor Registration Error on the Performance of PMHT

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## *Abstract-*

The Probabilistic Multi-hypothesis Tracking (PMHT) algorithm has been successfully applied to a simulated multi-static active sonar data set that contains a single constant velocity target in varying amounts of clutter [1]. The simulated data set in that study contained negligible registration error was therefore easily registered to a common frame of reference for use in a centralized tracking architecture.

Unknown sensor registration error can degrade the performance of any multi-static tracking algorithm that assumes that the measurement errors have zero mean. The registration error will appear as an unknown bias on the affected measurements and will degrade the data association stage of the tracker by increasing the probability that clutter detection will be confused for genuine target detections and vice versa. Moreover, if the registration error induced bias is large enough it may cause the tracking algorithm to misinterpret the track behavior (e.g., maneuvering instead of constant velocity). For many tracking algorithms this will cause the data association gates to be significantly expanded thereby increasing the probability that tracker will be drawn off target by clutter.

The purpose of the effort reported here is to investigate the effect of unknown registration error on the tracking performance of PMHT in a centralized architecture. This study quantifies the ability of PMHT to maintain track on a maneuvering contact while it is in close proximity to a fixed persistent clutter object in the search region to using a simulated multi-static active sonar data set created by MSTWG researchers at TNO. Specifically, the probability of track hold as a function of target SNR and registration error between two receivers will be evaluated via simulation using the TNO data set. In this way estimates of the maximum amount of registration error that can be tolerated will be computed over a range of relevant target SNR values.

*Index Terms-* Probabilistic Multi-hypothesis Tracker, multi-static active sonar, target tracking, registration error, centralized and distributed processing systems.

## 1. INTRODUCTION

Multi-static active sonar systems detect contacts of interest by transmitting coherent waveforms and detecting the echoes on one or more receiving sensors. Multi-static systems must

contend with the same confounding difficulties that mono-static systems face (e.g., clutter and unpredictable contact maneuvers) and some additional issues: aspect dependent variability in observed target SNR by different receivers, limitations on sensor coverage and overlap, imperfect sensor registration, and choice of processing architecture. When a target of interest is in a region where its echoes are detectable by more than one receiver it can, in general, be better tracked by combining the measurements from all sensors if the relative locations and orientations of the sensors is accurately known. However, if there is significant registration error between the sensors then the tracking performance will be degraded.

Considerable effort has been applied to develop methods that estimate any bias in the measurements from different sensors, [2] and [3]. Although these methods can be reasonably applied to low clutter situations involving fixed sensors that provide a high scan rate (e.g., radar systems), multi-static active sonar systems typically involve moving sensors, high clutter and a low scan rate. Thus a tracking method that is robust to registration errors remains desirable.

The analysis presented here utilizes a centralized processing architecture where the measurements (i.e., clustered echo detections) are registered to a common frame of reference and synchronized. Figure 1 illustrates the fundamental cycle of a centralized tracking architecture that performs sequential updates to a set of tracks using registered synchronized measurements from two sensors.

In this analysis an implementation of PMHT based on the centralized architecture depicted in figure 1 is used to evaluate track hold performance as a function of sensor registration error and target SNR. PMHT is designed to estimate a sequence of target states over a batch of measurements when the data is fundamentally incomplete; when the measurements contain no information on their origin (e.g., target or clutter). The derivation of the original PMHT algorithm is well described in [4] and is based on the so called independent assignment model; each measurement has some non-zero prior probability of being from any one of the targets present independent of the origin of all the other measurements.

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Under this assignment model it is entirely possible for all of the measurements to originate from any one of the targets but that hypothesis is almost always far less likely than more sensible assignments.

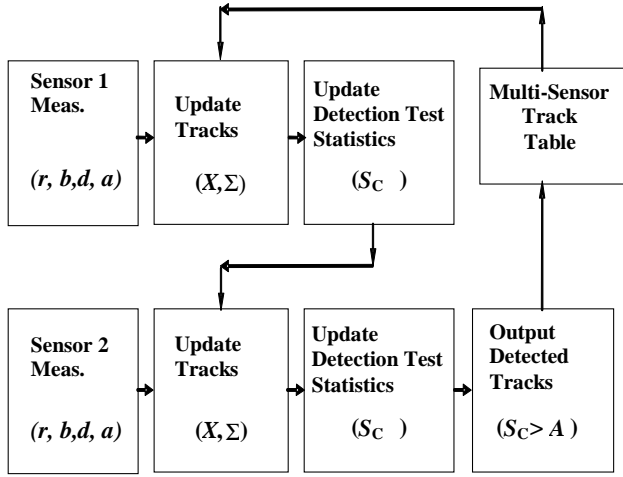


Figure 1. Centralized Multi-Sensor Tracking and Detection

The advantage of the independent assignment model is that when it is used in conjunction with the Expectation Maximization method it avoids having to enumerate a large number of candidate measurement assignment hypotheses and instead only requires the calculation of the posterior probabilities that the  $r$ 'th measurement at time  $t$  originated from target  $s$  as

$$w_{str} = \frac{\pi_s N(\mathbf{z}_{rt}; \mathbf{x}_{ts}, \mathbf{R}_{ts})}{\sum_{m=1}^M \pi_m N(\mathbf{z}_{rt}; \mathbf{x}_{tm}, \mathbf{R}_{tm})} \quad (1)$$

where  $\pi_i$  is the prior probability that a measurement originated from the  $i$ 'th target being tracked and  $M$  is number of targets. In [1] and [4] the above formula is modified to employ amplitude information and account for uniformly distributed clutter

$$w_{str} = \frac{\pi_s f_1(a_{rt}) N(\mathbf{z}_{rt}; \mathbf{x}_{ts}, \mathbf{R}_{ts})}{\frac{\pi_0}{V} f_0(a_{rt}) + \sum_{m=1}^M \pi_m f_1(a_{rt}) N(\mathbf{z}_{rt}; \mathbf{x}_{tm}, \mathbf{R}_{tm})}, \quad (2)$$

where  $V$  is the volume of the association gate, and  $f_0(a_{rt})$  and  $f_1(a_{rt})$  are the distributions for the echo amplitudes for clutter and target respectively. In this study the thresholded target echo amplitudes are Rayleigh distributed;

$$f_1(a) = \frac{\pi a}{2 P_D (1 + \theta)} e^{-\pi a^2 / 4(1 + \theta)} \text{ for } a > \tau. \quad (3)$$

and the clutter distribution is a thresholded unit mean Rayleigh;

$$f_0(a) = \frac{\pi a}{2 P_{fa}} e^{-\pi a^2 / 4}, \text{ for } a > \tau. \quad (4)$$

As described in [5] the basic PMHT algorithm amounts to iterating the following three steps:

1. Compute the association weights,  $w_{str}$ , for each measurement and target at each time step in batch.
2. Using the weights compute a measurement centroid and associated error covariance matrix (a.k.a. the synthetic measurement and covariance) for each target at each time step in the batch.
3. Update the track (i.e., the batch sequence of state estimates) for each target with a Kalman smoother on the synthetic measurements and error covariance matrices.

## 2. PURPOSE

This study quantifies the effects of sensor registration error and reduction in target SNR on the ability of PMHT to hold track on a maneuvering contact in close proximity to a fixed persistent clutter object to determine the ranges of conditions that provide acceptable tracking performance. The cases involving high registration error or reduced target SNR are of particular interest. Specifically, the probability of track hold as a function of sensor registration error and observed target SNR will be estimated for a system of two active sonar sources and receivers where both receivers produce monostatic and bistatic detections. The registration error will be modeled as a fixed position error between the two receivers that essentially induces biases in the measurements from most source/receiver combinations. In this way the values of registration error and target SNR that provide acceptable track hold will be identified allowing multi-static systems engineers to determine the suitability of such a design for various applications.

In order to focus on the effects of clutter density and registration bias the tracking conditions are assumed to be ideal in all other respects: perfect data normalization, independent and identically distributed zero mean measurement errors (except for registration error induced bias) with known covariance and a benign environment with identical interference level, propagation loss and target strength at all sensors.

## 3. THE SIMULATION

The simulated multi-static active sonar data set used in this analysis was constructed by members of the Multi-Static

tracking Working Group (MSTWG) at TNO using the Simulator of Non-acoustics and Acoustics (SIMONA) program [6]. The scenario contains two sources, two towed receivers, one moving target and two persistent fixed sources of clutter (i.e., false target detections) as shown in Figure 2. Both sources transmit once per minute and each source and receiving platform maintains a known constant velocity throughout the entire 180 ping scenario. This multi-static system generates two distinct mono-static combinations of source and receiver and two bi-static combinations. Figure 2 shows the ground truth of the simulated data set. The moving target proceeds from west to east in a zigzag trajectory of four legs at a speed of 7 knots. The two sources and receivers follow a heading of 90 degrees at a speed of 5 knots.

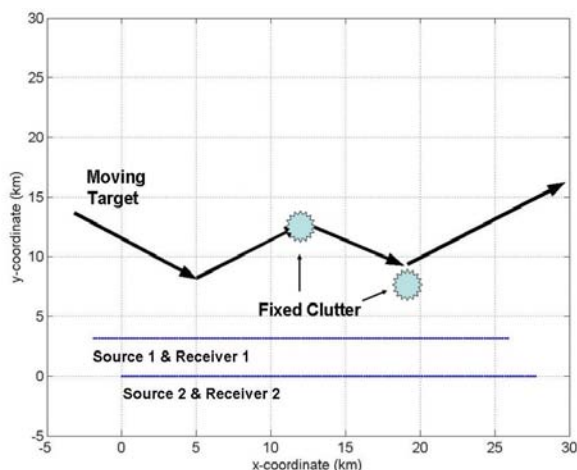


Figure 2. Ground Truth for Simulated Data Set.

The contact level measurements are sorted according to source/receiver combination and can be register to a common frame of reference. The data contains a substantial amount of unit mean Rayleigh clutter thresholded at 12dB with spatial density  $\lambda=1.0e-05$  detections/m<sup>2</sup>. Figure 3 is a plot of the simulated multi-static detections thresholded at 17db to reduce clutter.

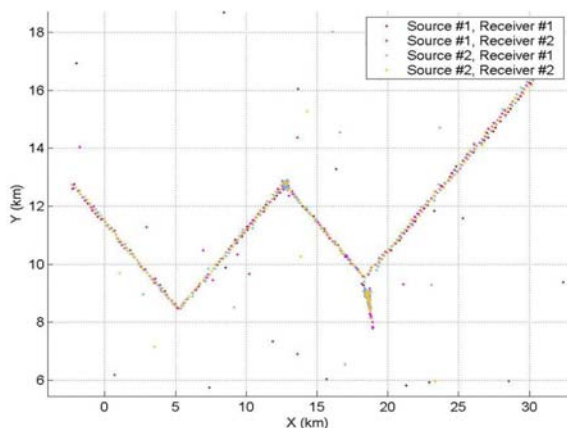


Figure 3. Simulated Data Set thresholded at 17dB

The detections of the moving and fixed contacts are clearly evident and are color coded according to which combination of source and receiver produced the detection. The original SNR of the moving target and the first clutter point are 18db while the SNR of the second clutter point is 28dB. In the course of this investigation only the SNR of the moving target is varied by recalculating the amplitudes of the relevant clusters. The full collection of simulated clutter in this data set is thresholded at 12db. This same threshold was used for all values of target SNR and registration error in the simulation runs.

The data in Figure 3 is clearly well registered. As part of this study sensor registration error is introduced as an error in the relative separation in the horizontal dimension of the two receivers; errors in the vertical dimension were not considered. While this choice may seem arbitrary, it is intended to approximately model the effect of a slight but unknown difference in the speeds of the receivers. Figure 4 illustrates the effect of an error of 1000 meters between the true and assumed horizontal position of the second source and receiver. For the moving target note that the monostatic detections for the first receiver are unaffected by the registration error, the bi-static detections of the first receiver are moderately biased and all of the detections for the second receiver are severely biased. Moreover, the bistatic detections for the fixed clutter objects appear to move over time in addition to being biased.

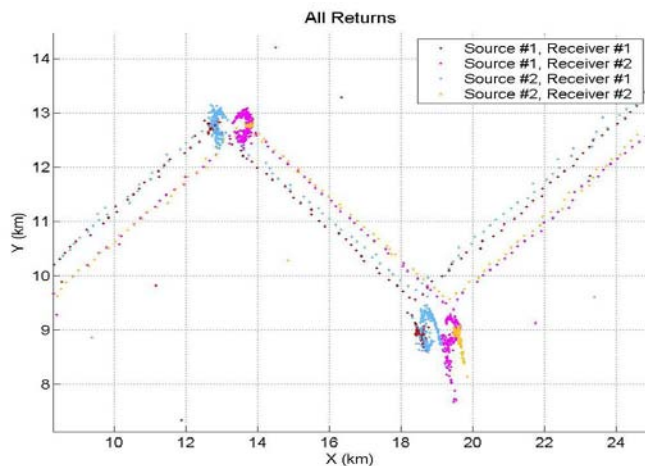


Figure 4. Simulated Data with 1000 meters of sensor registration error and thresholded at 17dB

In this study the track hold performance of PMHT is analyzed by varying the observed SNR of the moving target and by adding varying amounts of registration error to the TNO simulated multi-static data set. The implementation of PMHT applied here incorporates a piecewise linear white noise acceleration model with  $\sigma = 0.01\text{yd/sec}^2$  and uses a measurement error covariance matrix based on a range error standard deviation of 150 meters and a bearing error standard deviation of 1.5 degrees. A fixed value of 15dB is used for the target SNR parameter in the model for target echo

amplitude given by equation (3). At each update cycle PMHT estimates a sequence of target states for the most recent 15 ping cycles where each ping cycle produces 4 distinct scans (two mono-static and two bi-static) of measurements. The data association gates in this implementation of PMHT were fixed circular regions having radius 2500 meters; a track on the target had to be off by more than 2500 meters to avoid having the target detections in the association gate.

For values of target SNR less than 18 dB samples of an appropriate Rayleigh pdf are generated and thresholded at 12dB; samples less than 12dB are reset to zero and are then treated as missed detections and not used in PMHT. In this way the average values of the target amplitudes are reduced as well as the probability of target echo detection. All target tracks are initialized with ground truth to eliminate any effects of inaccurate track initialization. For each combination of registration error and target SNR 500 Monte Carlo runs were conducted and tracking performance for the moving target tabulated. The track on the moving target was declared lost whenever the current state estimate was more than 3000 meters from the ground truth

#### 4. RESULTS

The observed track hold percentage for the track on the moving target for each combination of registration error and target SNR are shown in Table 1. The track hold percentage is the average portion of the entire data set for which the track on the moving target remained within 3000 meters of ground truth. The track hold percentage can be converted to the average number of pings by multiplying by the total number of pings in the data set (i.e., 180 pings).

Track Hold Percentage							
		Registration Error					
		0 m	50 m	100 m	250 m	500 m	1000 m
Target SNR	18.0	100%	100%	100%	100%	91%	68%
	17.0	94%	91%	86%	81%	69%	55%
	16.0	88%	86%	81%	77%	53%	34%
	15.0	51%	48%	46%	40%	26%	17%
	14.0	23%	20%	18%	11%	5%	4%
	13.0	8%	3%	2%	1%	1%	1%

Table 1. Track hold percentage achieved by PMHT on the TNO simulated data set as a function of registration error and target SNR.

As can be expected, the track hold percentages in table 1 exhibit the consistent trends of increasing as registration error decreases and as target SNR increases. A detailed review of some of the simulation runs indicates that the vast majority of tracks on the moving target that held track through the first maneuver (see figure 2) but not the entire run lost track in the vicinity of one of the two fixed clutter objects at the second or third maneuver. This behavior appeared to be highly correlated with that of the tracks on the fixed clutter objects; if

the clutter tracks held on their respective objects then PMHT was usually able to hold track on the moving contact as well. However, some of the tracks on the moving contact lost due to extended dropout of the target detections caused by the random sampling of the relevant Rayleigh pdf.

#### 5. CONCLUSIONS

The results in the preceding section clearly show that the track hold performance of PMHT is robust to registration error provided the target is presenting adequate SNR. For target SNR values at or near the detection threshold (i.e., 12dB) the track hold ability of PMHT is much more sensitive to registration error. It should be pointed out that the probability that an echo from a target presenting an average SNR of 13dB will be above a detection threshold of 12dB is 0.53. Tracking a target that is below the detection threshold almost half of the time in the presence of significant clutter is a daunting challenge for any tracking method. The shaded entries in table 1 define a boundary above which PMHT provided acceptable track hold performance (i.e., greater than 80%).

The results in table 1 should be interpreted in the context of the capability of modern sensors many of which are equipped with GPS. Systems utilizing such sensors are unlikely to experience more than 100 meters of positional registration error. Although GPS is useful for reducing, if not eliminating, positional errors small errors in sensor orientation (e.g., 3 degrees) can produce large biases in the data especially for targets at extended ranges. Moreover, it is often impossible to arrange the sensors in a way that will provide high SNR detections on all contacts. Thus tracking methods that are robust to registration error and mismatch to assumed target SNR are desirable.

PMHT offers reasonably robust multi-target multi-static tracking performance, computational efficiency, and system flexibility; it can be implemented in either distributed or centralized architectures and combined with almost any track management logic. Appropriate track initialization methods have been presented in [7]. PMHT is a viable multi-target tracking method and it should be considered for use in fielded multi-static active sonar systems.

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#### REFERENCES

- [1] C. Hempel, "Probabilistic Multi-Hypothesis Tracking for Distributed Active Sonar," Proceeding of the 9<sup>th</sup> International Conference on Information Fusion, Florence Italy, July, 2006.
- [2] M. P. Dana, "Registration Techniques for Multiple Sensor Surveillance," Hughes Aircraft Co. Technical Report A931392,
- [3] S. Dhar, "Application of a Recursive Method for Registration Error Correction in Tracking with Multiple Sensors," Proceedings of the American Control Conference, San Francisco, CA, June 1993, WP9-15:30.

- [4] R. Streit, T. Luginbuhl, "*Probabilistic Multi-Hypothesis Tracking*," NUWC-NPT Technical Report 10,428, Naval Undersea Warfare Center Division, Newport, RI, 15 February 1995.
- [5] P. Willett, "*The Pedestrian PMHT*," Proceeding of the Fifth International Conference on Information Fusion, Annapolis, MD, July 8-11, 2002.
- [6] P. A. M. de Theije, H. Groen "*Multi-Static Sonar simulations with SIMONA*," Proceeding of the 9<sup>th</sup> International Conference on Information Fusion, Florence Italy, July, 2006.
- [7] C. Hempel, "*Track Initialization for Multi-Static Active Sonar Systems*," Proceeding of Oceans '07 IEEE Aberdeen, Aberdeen, Scotland, June 2007.